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Possible existence of faster-than-light phenomena for highly accelerated elementary particles

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The possible existence of faster-than-light(FTL) particles, which are forbidden by the known laws of physics, have been studied by many physicists. But the existence of such particles has not been confirmed by the experiments. This paper shows that faster-than-light phenomena can be permitted for highly accelerated elementary particles if they have very small mass compared to that of the electron.

Keywords: faster-than-light particles.

Symbols

ψ : wave function of the moving particle
α : proper acceleration of the particle
c : velocity of light
v : particle velocity
v_* : velocity of the particle in FTL state
P : momentum of the particle
E : energy of the particle
h : Planck's constant
$\hbar$: Planck's constant divided by 2π
m_0 : rest mass of the particle
m_* : absolute value of the FTL particle's rest mass
T : penetration probability of particles through the light barrier
λ_{co} : Compton wavelength of the particle
L : size of the atomic nucleus
Λ_0 : traveling distance of the particle in FTL state

Introduction

From the relativistic formula for kinetic energy, ordinary particles are confined in an infinite well of the velocity of light, so it is considered that faster-than-light(FTL) phenomena have no possibility of existence. Contrary to this conclusion, many physicists studied the possibility of FTL particles, which are called tachyons by G.Feinberg[1].

They have been searched for by various experiments[2-4], but most of them were negative to their existence.

The purpose of this paper is to show the possible existence of FTL phenomena for highly accelerated elementary particles in the quantum domain.

Wave Equation for The Uniformly Accelerated Particle

For the particle moving along the coordinate x, the wave function ψ can be shown as[5]

$$\psi(x,t) = \int_{-\infty}^{+\infty} g(k) \exp\left[i(kx - \omega t)\right] dk \quad , \qquad (1)$$

where k is a wave number, g(k) is a distribution function, ω is an angular frequency of the moving particle and t is a coordinate time.

For the uniformly accelerated particle, the proper acceleration α can be defined as follows:

$$p = m_0 \alpha (t - t_0) \quad , \qquad (2)$$

where P is the momentum of the particle, m_o is its rest mass and $(t- t_o)$ is the accelerated time interval of the particle.

By the special theory of relativity, the travelling distance x of the particle can be expressed as

$$x = \frac{c^2}{\alpha} \cosh\left(\frac{\alpha \tau}{c}\right) \quad , \qquad (3)$$

where c is the velocity of light and τ is a proper time .

By using the coordinate time t instead of τ, Eq.3 can be rewritten as[6]

$$x = \frac{c^2}{\alpha}\left[\left(1 + \frac{\alpha^2}{c^2} t^2\right)^{1/2} - 1\right] \quad . \qquad (4)$$

Substituting Eq.(4) into Eq.(1), the wave function of the particle can be obtained as

$$\psi(t)=\int_{-\infty}^{+\infty} g(k)\exp\left[i\left(kc^2(\mu-1)/\alpha-\omega t\right)\right]dk \quad , \tag{5}$$

where

$$\mu=\left(1+\frac{\alpha^2}{c^2}t^2\right)^{1/2} \quad . \tag{6}$$

From Eq. 4, the velocity of the particle can be written as $v=\alpha t/\mu$ [6], then the time derivative of ψ becomes

$$\frac{d\psi}{dt}=i(kv-\omega)\psi \quad . \tag{7}$$

The momentum P of the particle and its energy E can be shown using $\hbar$, the Planck's constant divided by 2π, as

$$P=\hbar k, \qquad E=\hbar\omega \quad . \tag{8}$$

Substituting them into Eq. 7, the wave equation for the moving particle becomes

$$\frac{d\psi}{dt}=\frac{i}{\hbar}(Pv-E)\psi \quad . \tag{9}$$

By the relativistic expressions of the momentum and the energy for the moving particle, which are given by[7]

$$P=\frac{m_0 v}{\left(1-v^2/c^2\right)^{1/2}} \quad , \tag{10}$$

and

$$E=\frac{m_0 c^2}{\left(1-v^2/c^2\right)^{1/2}} \quad . \tag{11}$$

The derivative with respect to the velocity of the particle can be given by

$$\frac{d\psi}{dv}=\frac{i}{\hbar}(Pv-E)\psi\bigg/\frac{dv}{dt} \quad , \tag{12}$$

where dv/dt is

$$\frac{dv}{dt}=\alpha\left(1-v^2/c^2\right)^{3/2} \quad . \tag{13}$$

From Eqs. 12 and 13, the wave equation for the moving particle can be written as

$$\frac{d\psi}{dv}=-i\frac{m_0c^4}{\alpha\hbar}\left(\frac{1}{c^2-v^2}\right)\psi \quad . \tag{14}$$

As shown in Fig.1, ordinary particles are confined inside the infinite well of energy (region Ⅰ) and they have no possibility of existence in the FTL region (region Ⅱ).
But we consider the particle velocity including imaginary values, the accelerated particle can penetrate through the light barrier avoiding the singular point C ,as shown in Fig.2, by tunneling effect.
By applying the WKB approximation[8] for Eq.14 to estimate the penetration probability of the particle through the light barrier, we have

$$\log\left(\frac{\psi(v_*)}{\psi(v_0)}\right) \approx -i\frac{m_0 c^4}{\alpha\hbar}\int_{v_0}^{v_*}\frac{dv}{c^2 - v^2} \quad , \tag{15}$$

where v_o is the original particle velocity and v_* is the velocity of the particle in FTL state.
The only way that P and E are real above the velocity of light, the rest mass of the particle in FTL state is required to be imaginary, then Eq.15 becomes

$$\log\left(\frac{\psi(v_*)}{\psi(v_0)}\right) \approx i\frac{m_0 c^3}{2\alpha\hbar}\log\left(\frac{c - v_0}{c + v_0}\right) - \frac{m_* c^3}{2\alpha_*\hbar}\log\left(\frac{v_* - c}{v_* + c}\right) \quad , \tag{16}$$

where m_* is an absolute value of the FTL particle's rest mass and α_* is the proper acceleration of the FTL particle that satisfies $|\alpha_*| = |\alpha|$.
As the absolute value of an exponent of the 1st term on the right side of Eq.16 becomes unity with regardless to the value of v_o and α_* in the 2nd term must have negative value to satisfy that $|\psi(v_*)/\psi(v_0)| = 0$ at v_*=c (i.e. the FTL particle doesn't exist at the velocity of light or it has a rest mass),the penetration probability *T* of the particle can be given by

$$T = \left[\frac{\psi(v_*)}{\psi(v_0)}\right]^2 \approx \exp\left[\frac{m_* c^3}{\alpha\hbar}\log\left(\frac{v_* - c}{v_* + c}\right)\right] \quad . \tag{17}$$

By using this equation, the probability for highly accelerated particles which can penetrate through the light barrier can be estimated.

Possible Existence of FTL Particles

By the uncertainty principle of momentum, $\Delta P \cdot \Delta x \sim h$, the following formula for the particle moving inside the atomic nucleus can be obtained as

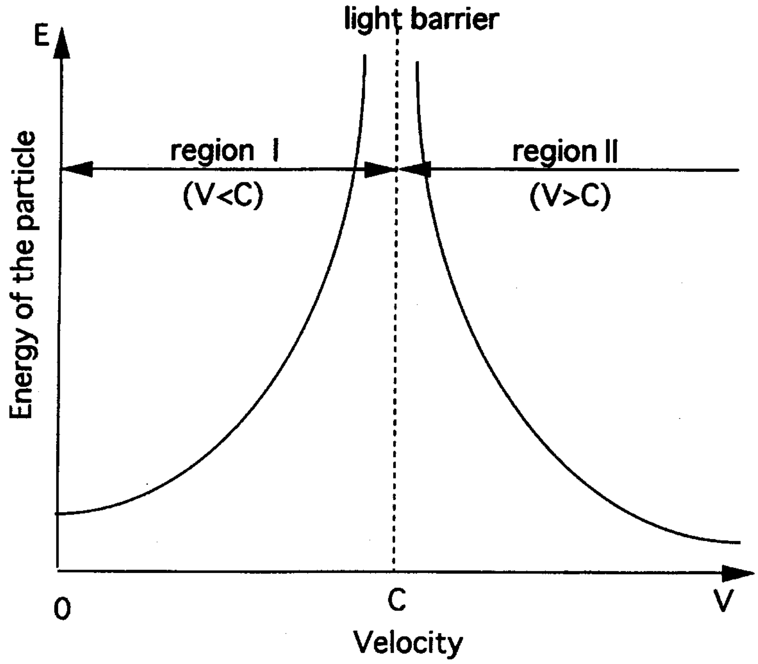

Fig. 1. Infinite well of energy for the moving particle.

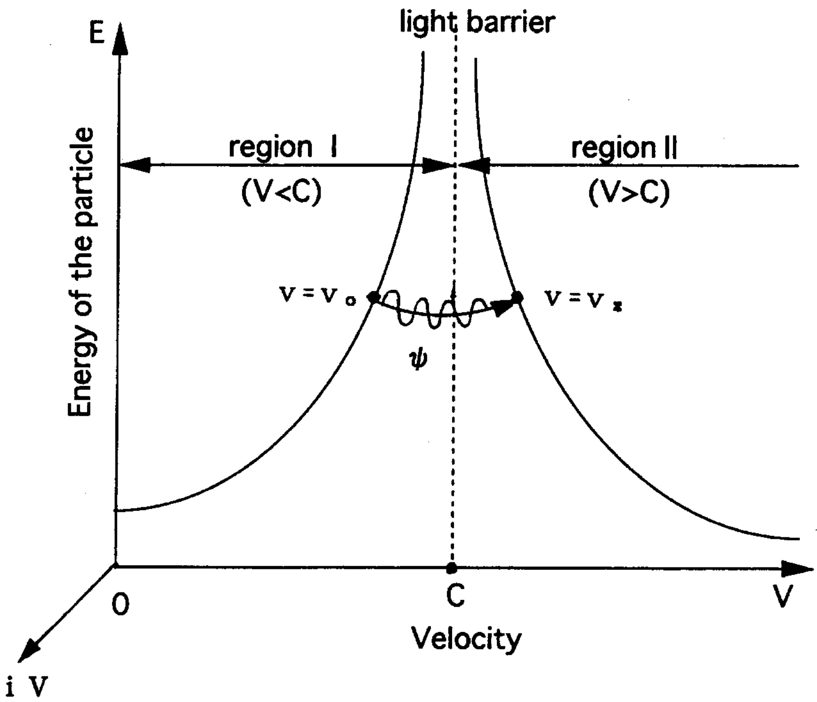

Fig. 2. Particle which penetrates through the light barrier.

$$\frac{m_0 v_0}{\left(1 - v_0{}^2 / c^2\right)^{1/2}} \approx \frac{h}{L} \quad , \tag{18}$$

where L is the size of the nucleus.
Then the velocity of the particle becomes

$$v_0 \approx \frac{c}{\left[1 + \left(m_0 c L / h\right)^2\right]^{1/2}} \approx c - \frac{c}{2}\left(\frac{m_0 c L}{h}\right)^2 \quad , \tag{19}$$

when the following condition is satisfied:

$$\frac{m_0 c L}{h} << 1 \quad . \tag{20}$$

From Eq.19, it is seen that light particles satisfying Eq.20 are highly accelerated inside the atomic nucleus and almost reach the velocity of light. Supposing that energy of the penetrated particle beyond the light barrier is conserved, the velocity of the particle in FTL state becomes

$$v_* = \left(2c^2 - v_0^2\right)^{1/2} \quad , \tag{21}$$

if m_* equals m_o .
By using the Compton's wavelength of the particle, which is defined as

$$\lambda_{co} = \frac{h}{m_0 c} \quad , \tag{22}$$

the difference of velocity between the FTL particle and the light becomes

$$v_* - c \approx \frac{c}{2}\left(\frac{L}{\lambda_{co}}\right)^2 \quad . \tag{23}$$

From Eqs.19 and 21, we have

$$v_* + c \approx 2c \quad . \tag{24}$$

Then the probability of particles transferred into FTL state can be given by

$$T \approx \exp\left(\frac{2m_0 c^3}{\alpha\hbar} \log\left(\frac{L}{2\lambda_{co}} \right) \right) \quad . \tag{25}$$

By using Eq.4, the proper acceleration of a particle in the atomic nucleus can be approximated as

$$\alpha \approx \frac{c^2}{L}\left[\left(1 + \frac{v_0{}^2}{c^2} \right)^{1/2} - 1 \right] \approx 0.414 \frac{c^2}{L} \quad . \tag{26}$$

Then T can be written as

$$T \approx \exp\left(9.66 \frac{\pi L}{\lambda_{co}} \log\left(\frac{L}{2\lambda_{co}} \right) \right) \quad . \tag{27}$$

By the uncertainty principle of momentum and the difference of momentum ΔP, which is given by

$$\Delta P = \left| P_* - P_0 \right| = \frac{m_0 (v_* - v_0)}{\left(1 - v_0{}^2 / c^2 \right)^{1/2}}$$

$$\approx \frac{m_0{}^2 c^2 L}{h} = \frac{hL}{\lambda_{co}{}^2} \quad , \tag{28}$$

where P_o is the original momentum of the particle and P_* is the momentum of the particle in FTL state, the travelling distance Λ_o of the particle in FTL state can be roughly estimated as

$$\Lambda_0 \approx h / \Delta P = \frac{\lambda_{co}{}^2}{L} \quad . \tag{29}$$

Fig.3 is the calculated result of the relation between the probability for the particle transferred into FTL state vs. its original rest mass m_o and Fig.4 is the relation between the traveling distance in FTL state Λ_o vs. the particle's rest mass m_o, supposing that the size of the atomic nucleus is 10^{-14} m.

In Fig.4, possible values of the electron neutrino rest mass are also indicated by symbols ν_1, ν_2 and ν_3, which are shown in Table.1.

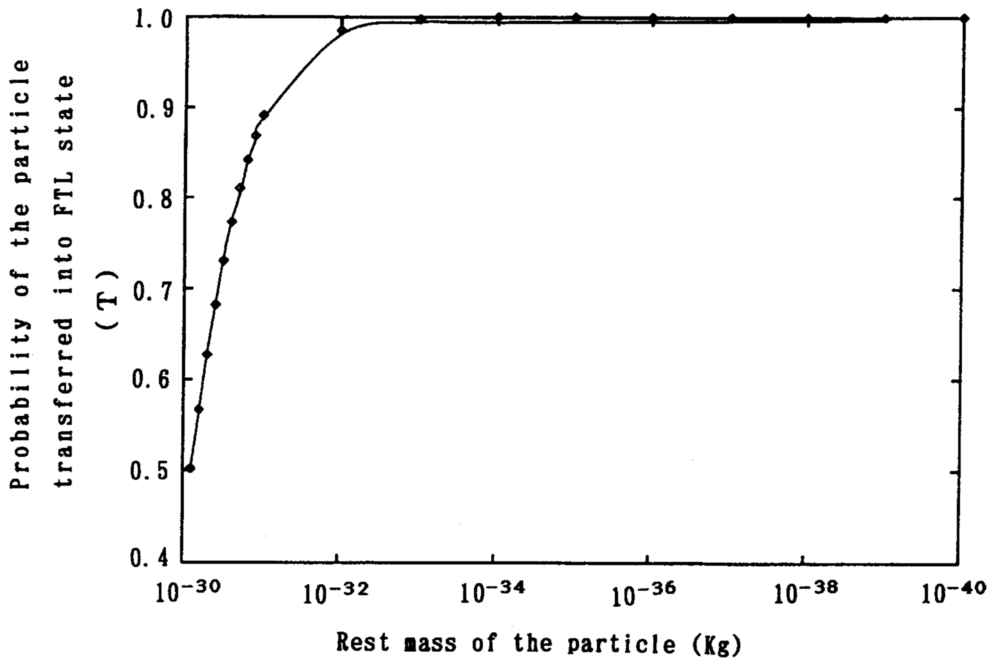

Fig. 3. Probability of the particle transferred into FTL state vs its original rest mass.

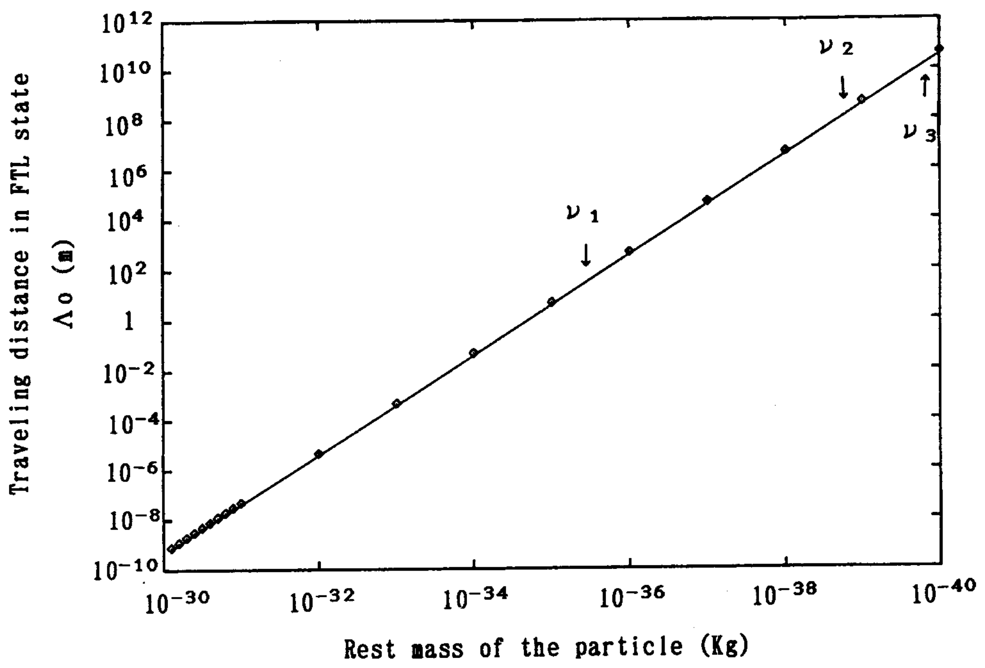

Fig. 4. Travelling distance of the particle in FTL state vs its original rest mass.

From Fig.3, it is seen that the light particle created inside the atomic nucleus, which has the non-zero rest mass less than 10^{-32} Kg, has the probability of almost unity to transfer into FTL state.

Table.1 Possible values of the rest mass of an electron neutrino

Symbol	m_o (Kg)	Ref.
ν_1	$2\sim7\times10^{-36}$	9
ν_2	1.2×10^{-39}	10
ν_3	1.5×10^{-40}	11

Forward reported in his scientific paper[12] that the electron neutrino and the muon neutrino have been experimentally observed as tachyons which have imaginary rest mass. If the rest mass of the neutrino emitted from the atomic nucleus is less than 10^{-32} Kg ,its traveling distance of the particle in FTL state becomes over 1m and they have possibility to be experimentally detected as tachyons. If the rest mass of the neutrino is less than 10^{-41} Kg, the value of Λ_o becomes over 10^{12} m inferred from Fig.4. Supposing that imaginary-mass particle would not interact with real-mass particles, solar neutrinos can hardly be detected on the Earth because the distance between the Earth and the Sun is about 1.5×10^{11}m and most of solar neutrinos reach the Earth in FTL state.

Conclusion

In this paper, the possibility of the existence of FTL phenomena in the quantum domain is discussed. The theoretical analysis gives the result that FTL phenomena could exist for light particles like electron neutrinos created inside the atomic nucleus if they have non-zero small rest mass.

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Appendix (Experiment evidence for FTL phenomena)

(A) Measurement of the electron neutrino rest mass carried out by measuring the end point of the energy spectrum of the electron emitted in the decay of tritium has shown that the square of the electron neutrino rest mass seems to be negative, which implies that the electron neutrino has an imaginary rest mass.

(1986) $-158 \pm 253 \ eV^2$, (1991) $-147 \pm 109 \ eV^2$,

which is not acceptable in the existing theories for the neutrino, but it suggests that electron neutrinos are tachyons. Similar results seem to be coming from measurements of the muon neutrino rest mass.

[A1]. R.L.Forward, Indestinguishable from Magic, Baen Books, N.Y.(1995)353-358

(B) Most tritium beta decay experiments has given negative value for m^2. Such results would be fairly conclusive evidence that the electron neutrino was a tachyon.

(1973) $-0.29 \pm 0.90 \ MeV^2$. (1980) $-0.102 \pm 0.119 \ MeV^2$.

(1982) $-0.14 \pm 0.20 \ MeV^2 / c^4$. (1984) $-0.163 \pm 0.080 \ MeV^2 / c^4$

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